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# Structure Properties of Yrast Superdeformed Bands in the Mass Region Around Gd-144

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ABSTRACT: Eight yrast superdeformed (SD) bands around the doubly magic SD nucleus <sup>144</sup>Gd (Z=64, N=80) are analyzed using the alignment concept and energy reference. The bandhead spins have been assigned from the comparison of the experimental dynamical moments of inertia  $J^{(2)}$  with a theoretical version of Harris three parameters expansion in even powers of angular frequency . The aligned angular momenta are calculated with respect to adopted rigid rotor moment of inertia reference. A band crossing at  $\hbar\omega$  0.45 MeV for the yrast SD band of <sup>144</sup>Gd is reproduced due to aligned of two protons in the  $i_{13/2}$  orbital; the gain in alignment is about 6.5  $\parallel$ . The incremental alignment has been calculated relative to the yrast SD band of <sup>143</sup>Eu built on the proton  $i_{13/2}$  configuration. Quantitatively very good results for gamma ray transition energies, rotational frequencies, kinematic  $J^{(1)}$  and dynamic  $J^{(2)}$  moments of inertia are obtained. The systematic variation of  $J^{(1)}$  and  $J^{(2)}$  are investigated, flat  $J^{(2)}$  is observed for the N=80 isotones.

### I. INTRODUCTION

Since the original experimental discovery of the first discrete line superdeformed (SD) rotational band in the nucleus <sup>152</sup>Dy [1] in 1986, the investigation of superdeformation at high angular momenta remains one of the most interesting and exciting topics of nuclear structure. While much theoretical and experimental work has been done over the past several years on SD nuclei, a new era in the study of nuclei at high angular momenta has begun with the recent construction of Gammasphere, Eurogam and Gasp gamma ray spectrometer arrays. Nowadays a great number of SD bands have been observed in various mass regions [2,3]. Theoretically the nucleus  $^{144}$  Gd ( Z=64, N=80) was expected to be a good candidate for the observation of superdeformation in the mass region A 140 [4]. Till 1993 several experiments was carried out, but none of them was successful to search for SD bands in this nucleus. In 1994 S.Lunardi et al [5] found experimentally SD bands in the nucleus <sup>144</sup> Gd.

Despite the impressive success of experimental information about SD rotational bands, there remains several problems not yet completely solved. Among them are the uncertain assignment of the spin and parity of SD rotational bands. To date several theoretical related approaches to assign the spins of SD bands in terms of their observed gamma-ray transition energies have been proposed [6-17]. For all approaches an extrapolation fitting procedure was used. An interesting aspect of SD studies has been the identical band phenomenon [18,19], whereby SD bands with very nearly identical energies have been observed in different nuclei. Several groups tried to understand this phenomenon in framework of phenomenological and semi phenomenological methods [20-22]. Stephens *et al* [23] have used the incremental alignment, which represent the aligned spin of SD bands relative to one another to compare the SD bands in neighboring nuclei [24,25]. The advantage of incremental alignment is that it depends only on gamma ray transition energies, and not upon the unknown spins.

The main purpose of the present work is to predict the spins of the bandheads of our selected SD bands around <sup>144</sup> Gd and to examine the main properties like the behavior of moments of inertia, the alignment and the incremental alignment of our SD rotational bands by using the Harris model. The paper is arranged as follows: In section 2, the phenomenological Harris model for fitting SD rotational bands are described for assign the spins. Using the cranked shell model the aligned angular momentum is derived in section 3. Additional information about the structure of SD bands can be obtained by comparing the transition energies directly to those in other bands by using the incremental alignment in section 4. Numerical calculations for the characteristic features of our selected SD bands are performed in section 5. Conclusion and remarks are given in section 6.

## **II. SPIN ASSIGNMENT IN SD BANDS USING HARRIS EXPANSION**

The nuclear rotational energies within a band exhibit a smooth dependence on spin and may therefore very easily be described with power series expansions in terms of spin I(I+1) or in terms of rotational angular velocity . In Harris expansion [26] of cranked analysis, the rotational excitation energy E is given in terms of even powers of the angular frequencies up to  $^{6}$  as:

$$E = A\omega^2 + B\omega^4 + C\omega^6 \tag{1}$$

where the angular frequency is not directly observed quantity, but is derived from the observed rotational spectrum according to the canonical relations

$$\hbar\omega = \frac{dE}{d\hat{l}} \tag{2}$$

with

$$I = [I(I+1)]^{1/2}$$
(3)

where  $\widehat{I}$  is the intermediate nuclear spin.

The expansion coefficients A, B and C have the dimensions,  $\hbar^2 MeV^{-1}$ ,  $\hbar^4 MeV^{-3}$  and  $\hbar^6 MeV^{-5}$  respectively. The standard way to analyze SD bands, is to consider the dynamical moment of inertia  $J^{(2)}$  because it does not require any knowledge of the spin value which is not determined experimentally. Using the definition of dynamical moment of inertia, yield

$$\frac{J^{(2)}}{\hbar^2} = \left(\frac{d^2E}{d\hat{I}^2}\right)^{-1} = \frac{1}{\hbar}\frac{d\hat{I}}{d\omega} = \frac{1}{\hbar^2}\frac{1}{\omega}\frac{dE}{d\omega} = 2A + 4B\omega^2 + 6C\omega^4 \tag{4}$$

Integrating  $J^{(2)}$  with respect to  $\omega$  leads to expression for spin I as a function of

$$\hbar \hat{I} = \int d\omega J^{(2)} = 2A\omega + \frac{4}{3}B\omega^3 + \frac{6}{5}C\omega^5 - i_o$$
(5)

where io is the constant of integration (aligned spin)

Finally the kinematic moment of inertia  $J^{(1)}$  has the expression:

$$\frac{J^{(1)}}{\hbar^2} = \frac{I}{\hbar\omega} = 2A + \frac{4}{3}B\omega^2 + \frac{6}{5}C\omega^4$$
(6)

Experimentally, one can extract  $\hbar\omega$ , J(2) and J(1) by using the experimental interaband E2 transition energies as:

$$\hbar\omega(I) = \frac{E_{\gamma}(I+2-I) + E_{\gamma}(I \to I-2)}{\frac{4}{4}} \quad (MeV)$$
(7)

$$I^{(2)}(I) = \frac{4}{E_{\gamma}(I+2\to I) - E_{\gamma}(I\to I-2)} (\hbar^2 M e V^{-1})$$
(8)

$$J^{(1)}(I) = \frac{2I - 1}{E_{\gamma}(I \to I - 2)} (\hbar^2 M e V^{-1})$$
(9)

It is seen that, while  $J^{(1)}$  depends on the spin proposition,  $J^{(2)}$  does not.

### **III. ALIGNED ANGULAR MOMENTUM**

In cranked shell model (CSM), the aligned angular momentum (alignment) i is the difference in spin with respect to a chosen reference band at a fixed rotational frequency, it is defined as[27]

$$i_x(I_m) = I_x(I_m) - I_x^{ref}(I_m)$$
(10)

where  $I_m$  is the mean angular momentum and the angular momentum  $I_x(I_m)$  represent the projection of the total angular momentum on the axis of rotation x and is estimated as

$$I_x(I_m) = (I_m + \frac{1}{2})$$
(11)

The frequency dependent reference  $I_x^{ref}$  is taken with respect to rigid rotor as first approximation as

$$I_x^{ref} = J_o \omega(I_m) \tag{12}$$

In this case the rotational frequency can be extracted from the transition energies as

$$\hbar\omega(I_m) = \frac{dE(I)}{dI} = \frac{E(I_m+1)-E$$

where E(I) represent the excitation energy of the level with spin I.

If we assume for  $[I_x(I_{m+1}) - I_x(I_{m-1})]$  a constant value 2, we obtain a good approximation of  $\hbar\omega(I)$  by

$$\hbar\omega(I) = \frac{1}{2} [E(I+1) - E(I-1)]$$
(14)

(13)

### **IV. INCREMENTAL ALIGNMENT**

Additional information about the structure of a given SD band can be obtained by comparing the transition energies directly to those in other bands. The incremental alignment i is a convenient way in which to determine these relationships. This incremental alignment has the important advantage that it does not require knowledge of the spins of the states. The incremental alignment  $\Delta i$  is defined as [28]:

with

$$\begin{split} & \Lambda i = 2 \frac{\Delta E_{\gamma}}{\Delta E_{\gamma}^{ref}} \qquad (15) \\ & E_{\gamma} = E_{\gamma}^{A}(I+2) - E_{\gamma}^{B}(I) \qquad (16) \\ & E_{\gamma}^{ref} = E_{\gamma}^{B}(I+2) - E_{\gamma}^{B}(I) \qquad (17) \end{split}$$

In these expressions,  $\Delta E_{\nu}$  is obtained by subtracting the transition energy in a band of interest A from the closest transition energy in the reference SD band B and  $E_{\gamma}^{ref}$  is calculated as the energy difference between the two closest transitions in the SD band of the reference B. In our study the yrast SD bands in <sup>143</sup>Eu is chosen as the reference because of its approaches to doubly magic character (<sup>144</sup>Gd). Usually *i* values cluster around values zero for even-even nuclei with the same signature and  $\pm 1$  with different signature and 0.5 for odd-even nuclei relative to an even-even core in the strong coupling limit.

# V. NUMERICAL CALCULATIONS AND DISCUSSION

The spins of each SD band are determined by fitting the experimental dynamical moment of inertia  $\int_{exp}^{(2)}(i)$  with the expression calculated from the Harris three parameters formula. It has been argued that at zero rotational frequency, the aligned spin  $i_o$  is equal to zero. The optimized best expansion parameters A, B and C in question have been adjusted by using a computer simulated search program in order to

minimize the common definition of the root mean square (rms) deviation  $\chi$ , given by

$$= \left[\frac{1}{N}\sum_{i=1}^{N} \left|\frac{J_{exp}^{(2)}(i) - J_{cal}^{(2)}(i)}{J_{exp}^{(2)}(i)}\right|^{2}\right]^{1/2}$$

where N is the number of experimental data points entering into the fitting procedure.

Table (1) summarize the values of the bandhead spin  $I_0$ , the lowest transition energy  $E_{\gamma}(l_0 + 2 - l_0)$  and the adopted model parameters A,B and C obtained from the best fitting procedure. Using these values, the kinematic J<sup>(1)</sup> and dynamitic J<sup>(2)</sup> moments of inertia are calculated and plotted versus the rotational frequency  $\hbar\omega$  and compared to the  $J^{(2)}$  value obtained from the experimental transition energies by assuming three different spins  $I_0$ -2,  $I_0$  and  $I_0$ +2. Fig. 1 represents an example for  ${}^{143}$ Eu (SD1) at I<sub>0</sub>= 12.5, 14.5 and 16.5. We see that the best agreement is obtained for bandhead spin  $I_0=14.5$ , which is the predicted value from the theory. Also the energy relative to a reference is a tool that predict the bandhead spin. The standard choice of such a reference is the energy of pure rotator; this is shown in Figure (2). The agreement between the calculated transition energies and the experimental one for our selected SD bands are excellent (the experimental data are taken from Ref [3]).

The dynamic moments of inertia  $J^{(2)}$  are a convenient way of comparing the various SD bands since those do not require knowledge of the spins. The calculated kinematic  $J^{(1)}$  and dynamic  $J^{(2)}$  moments of inertia as a function of rotational frequency  $\hbar\omega$  are plotted in Figure (3). For <sup>144</sup>Gd, the results show a fairly sharp band crossing at a rotational frequency of  $\hbar\omega =$ 0.45 *MeV*, which is interpreted as a crossing of the  $\pi 6^0$ configuration with alignment of a proton pair  $\pi 6^2$ configuration.

Table 1: The adopted parameters used in the calculation for our selected yrast SD bands around <sup>144</sup>Gd. I<sub>o</sub> and J<sub>o</sub> represent the bandhead spin and the moment of inertia reference respectively. A, B and C are the Harris expansion coefficients resulting from the fitting procedure of the dynamical moments of inertia with the experimental ones.

Yrast SD	$E_{\gamma}(l_0 + 2 l_0)$	Io	Jo	А	В	C
Band	(KeV)	(ħ)	$(\hbar^2 \text{MeV}^{-1})$	$(\hbar^2 \text{MeV}^{-1})$	$(\hbar^2 \text{MeV}^{-1})$	$(\hbar^2 \text{MeV}^{-1})$
$^{142}$ Sm	679.7	23	69.4	33.9738	-1.8984	-0.3748
<sup>142</sup> Eu	699.7	25	72.2	30.3607	7.3534	-5.7301
<sup>143</sup> Eu	483.28	16.5	70.1	31.5603	5.3440	-3.9512
<sup>144</sup> Eu	878.6	34	80.0	30.3608	7.3535	-5.7302
$^{143}$ Gd	546.0	23.5	80.2	54.5144	-38.2451	24.5928
$^{144}$ Gd	802.8	22	70.1	29.7827	14.2092	-15.0674
<sup>145</sup> Gd	723.2	29.5	83.0	48.0206	-25.8455	17.5904
<sup>145</sup> Tb	627.1	20.5	69.39	30.9801	12.5395	-13.5799



Fig. 1. The calculated kinematic moment of inertia  $J^{(1)}$  of <sup>143</sup>Eu(SD1) is plotted against rotational frequency  $\hbar$  and compared to the  $J^{(2)}$  values obtained from the experimental transition energies assuming three different bandhead spins  $I_0 = 12.5, 14.5, 16.5$ .



**Fig. 2.** The upper panels represent the <sup>143</sup>Eu yrast SD calculated excitation energies relative to standard I(I+1) reference as a function of spin for three bandhead spin values  $I_0$ =14.5,  $I_0$ =16.5 and  $I_0$ =18.5. The rigid –rotator reference energy of 0.07I(I+1) has been subtracted to increase clarity of the plot. The lower panel represent the differences.

$^{142}$ Sm(SD1)		<sup>143</sup> Eu(SD1)		<sup>144</sup> Gd(SD1)		$^{142}$ Eu(SD1)	
ħ	i	ħ	i	ħ	i	ħ	i
(MeV)	(九)	(MeV)	(ħ)	(MeV)	(九)	(MeV)	(九)
0.3398	0.4144	0.24164	0.5610	0.4014	-5.1381	0.34985	0.7408
0.3697	0.3428	0.27318	0.3500	0.4231	-4.6593	0.38095	0.4954
0.3998	0.2504	0.30442	0.1598	0.4396	-3.8159	0.41140	0.2969
0.4300	0.1580	0.33557	-0.0234	0.4463	-2.2856	0.44315	0.0045
0.4602	0.0621	0.36621	-0.17130	0.4512	-0.6326	0.47355	-0.1903
0.4903	-0.0268	0.39656	-0.29920	0.4671	0.2562	0.50415	-0.3996
0.5206	-0.1331	0.42665	-0.40810	0.4900	0.6510	0.53470	-0.6053
0.5509	-0.2324	0.45645	-0.49710	0.5156	0.8529	0.56470	-0.7713
0.5814	-0.3526	0.48163	-0.5777	0.5423	0.9847	0.59355	-0.8543
0.6122	-0.4866	0.51562	-0.6453	0.5697	1.0605	0.62395	-1.0491
0.6431	-0.6311	0.54496	-0.7020	0.5973	1.1257	0.65395	-1.2151
0.7054	-0.9582	0.57433	-0.7608	0.6249	1.1945	0.68410	-1.3920
0.7372	-1.1651	0.60356	-0.8099	0.6528	1.2387	0.71420	-1.5652
0.7691	-1.3790	0.62284	-0.8624	0.6810	1.2619	0.74340	-1.6734
0.8014	-1.6171	0.66226	-0.9244	0.7089	1.3061	0.74700	-1.8628
0.8338	-1.8657	0.69135	-0.9636	0.7372	1.3187		
0.8664	-2.1281	0.72156	-1.0813	0.7659	1.3104		
0.8991	-2.4010	0.75151	-1.1812	0.7940	1.3406		
		0.78165	-1.2936	0.8225	1.3427		
		0.81200	-1.4212				
		0.84235	-1.5487				
		0.87315	-1.7078				
		0.90240	-1.7582				

Table 2(a): The aligned angular momentum $i$ (alignment) as a function of rotational frequency	ħe	for
$^{142}$ Sm, $^{143}$ Eu, $^{144}$ Gd and $^{142}$ Eu. The moment of inertia reference parameter J <sub>o</sub> is given in table	(1).	

# Table 2b: The same as in Table (2a) but for <sup>143</sup>Gd(SD1), <sup>145</sup>Gd(SD1), <sup>144</sup>Eu(SD1) and <sup>145</sup>Tb(SD1).

<sup>143</sup> Gd(SD1)		<sup>145</sup> Gd(SD1)		<sup>144</sup> Eu(SD1	)	<sup>145</sup> Tb(SD1	)
ħ	i	ħ	í	ħ	i	ħ	i
(MeV)	(九)	(MeV)	(ħ)	(MeV)	<u>(ħ)</u>	(MeV)	<u>(ħ)</u>
0.27300	2.6054	0.3616	0.4872	0.4393	-0.144	0.3135	-0.2572
0.30450	2.0791	0.3848	0.5616	0.4513	0.892	0.3439	-0.3632
0.33525	1.6129	0.4090	0.5488	0.4763	0.892	0.3735	-0.4171
0.36585	1.1588	0.4340	0.4780	0.5058	0.532	0.4030	-0.4676
0.39630	0.7167	0.4594	0.3698	0.5346	0.228	0.4322	-0.4938
0.42685	0.2666	0.4854	0.2118	0.5640	-0.120	0.4600	-0.4194
0.45655	-0.1153	0.6119	0.0123	0.5930	-0.444	0.4901	-0.5115
0.48605	-0.4812	0.5385	-0.1996	0.6224	-0.796	0.5196	-0.5585
0.51520	-0.8190	0.5655	-0.4365	0.6511	-1.088	0.5486	-0.5708
0.5441	-1.1368	0.5929	-0.7148	0.6816	-1.532	0.5775	-0.5727
0.59345	-1.4906	0.6201	-0.9683	0.7107	-1.856	0.6059	-0.5468
0.60280	-1.8445	0.6476	-1.2549	0.7393	-2.144	0.6356	-0.6077
0.63225	-2.2064	0.6753	-1.5499			0.6620	-0.4361
0.66190	-2.5843	0.7034	-1.8822			0.6935	-0.6219
0.69185	-2.9863	0.7320	-2.2560				
		0.7600	-2.5841				
		0.7881	-2.9123				



**Fig. 3.** Results of the dynamic moment of inertia  $J^{(2)}$  (closed circle) and kinematic moment of inertia  $J^{(1)}$  (open circles) as a function of rotational frequency  $\hbar_{1}$  of the yarst SD bands in N=79, 80, 81 isotones.







### Rotational Frequency ħ( (MeV)

**Fig. 4.** Alignment plot: alignment as a function of rotational frequency  $\hbar_{00}$  for yrast SD bands (a) For <sup>142</sup>Eu and <sup>143</sup>Gd with respect to a rigid rotor reference with moment of inertia ( $L_0 = 72.2 \ h^2 \ MeV^{-1}$ ) and ( $J_0 = 80.2 \ h^2 \ MeV^{-1}$ ) respectively (b) The same as in (a), but for yrast SD bands of <sup>142</sup>Sm ( $J_0 = 69.4 \ h^2 \ MeV^{-1}$ ), <sup>144</sup>Gd ( $J_0 = 70.1 \ h^2 \ MeV^{-1}$ ) and <sup>145</sup>Tb ( $J_0 = 69.39 \ h^2 \ MeV^{-1}$ ) (c) The same as in (a), but for yrast SD bands of <sup>144</sup>Eu ( $J_0 = 80 \ h^2 \ MeV^{-1}$ ) and <sup>145</sup>Gd ( $J_0 = 83 \ h^2 \ MeV^{-1}$ ). For comparison, also the data for the yrast SD band of <sup>143</sup>Eu ( $J_0 = 70.1 \ h^2 \ MeV^{-1}$ ) as a reference is included.

After the crossing the SD band has either a  $(\pi 6^2 \nu 7^0)$  or  $(\pi 6^2 \nu 7^1)$  configuration. Flat  $J^{(2)}$  are observed in the yrast SD bands of the isotones N=80 ( $^{142}$ Sm ,  $^{143}$ Eu and  $^{145}$ Tb ). The absence of band crossing can be explained as a blocking effect if at least one  $i_{13/2}$  proton orbital is populated in these nuclei. Actually a  $(\pi 6^1 \nu 6^4 \nu 7^0)$  configuration has been assigned to the SD in  $^{143}$ Eu. The configuration of the SD band in  $^{142}$ Sm is considered to have the same configuration that in  $^{143}$ Eu coupled to a hole in the i/2 - [541] Nilsson orbital. For  $^{145}$ Tb a  $\pi 6^1 \otimes (9/2 + [404])^2$  configuration has been suggested for this SD band. Hence in all three isotones, the  $i_{13/2}$  proton orbital is blocked so that the absence of

band crossing can be understood.  $J^{(2)}$  for  $^{145}Gd$  starts with value  $86\hbar^2~MeV^{-1}$  and then decreases smoothly with increasing rotational frequency  $\hbar\omega$  to a value of  $71\hbar^2~MeV^{-1}$  this yrast SD band built on the  $N{=}7$ ,  $j_{15/2}$  neutron orbital.

The yrast SD band in <sup>142</sup>Eu (configuration ( $\pi 6^1 \nu 6^3$ ) has an almost constant J<sup>(2)</sup> and extends over a large frequency range from 0.25 MeV to 0.7 MeV. In order to discuss the rotational properties of our eight yrast SD bands, the aligned angular momentum *i* are calculated and plotted in Figure (4) as a function of rotational frequency  $\hbar\omega$ .

The numerical values are listed in Table (2) and the adopted moment of inertia reference Jo for each band is given in Table (1). In <sup>144</sup>Gd, the gain in aligned angular momentum is about  $6\hbar$ , the reduction in gain is due to the weaker Coriolis interaction. At  $\hbar\omega \sim 0.45 \ MeV$  a sharp crossing is happen and the nucleus is driven to large deformation, this is interpreted as a crossing of the  $(\pi 6^0 \nu 7^0)$  configuration with the  $(\pi 6^2 \nu 7^0)$  configuration (alignment of a proton pair).

The incremental alignment *.i* is calculated and plotted against rotational frequency  $\hbar\omega$  in Figure (5). The numerical values are listed in Table (3). Calculations [29] predicted a large shell gap at N=80 and indicated that nuclei such as <sup>144</sup>Gd should have SD bands with deformation of  $\beta_2 = 0.5$ . The SD band in <sup>143</sup>Eu seems to be central to these eight bands, and we used it as a reference band.

ħω	i						
	<sup>142</sup> Eu(SD1)	$^{143}$ Gd(SD1)	$^{144}$ Eu(SD1)	$^{145}$ Gd(SD1)	$^{142}$ Sm(SD1)	$^{144}$ Gd(SD1)	<sup>145</sup> Tb(SD1)
0.2888		-0.0115					
0.3199		0.0048					0.5859
0.3508	0.9321	-0.0208			0.2793	4.2968	0.5437
0.3813	0.9711	-0.0237		-0.303	0.2299	3.7482	0.4803
0.4115	0.9863	-0.0176		-0.783	0.2187	2.8612	0.4311
0.4415	1.1073	0.0134	0.8489	-1.181	0.2248	1.3186	0.3758
0.4712	1.1522	0.0067	-0.3436	-1.512	8 0.2526	-0.3504	0.2392
0.5008	1.2219	-0.0054	-0.6631	-1.812	5 0.2827	-1.2902	0.2725
0.5302	1.3002	-0.0289	-0.6663	-2.060	3 0.3425	-1.7466	0.2743
0.5596	1.3438	-0.0589	-0.7024	-2.251	6 0.4041	-1.9967	0.2509
0.5889	1.3147	-0.0605	-0.7071	-2.448	5 0.4869	-2.1918	0.2165
0.6192	1.3924	-0.0522	-0.7182	-2.600	0 0.5898	-2.3098	0.1629
0.6475	1.4349	-0.0404	-0.7067	-2.712	5 0.6972	-2.4132	0.1907
0.6768	1.5015	-0.0247	-0.7672	-2.898	5 0.8140	-2.5684	-0.0178
0.7064	1.5127	0.0331	-0.6421	-2.893	0 0.9334	-2.552	0.1423
0.7365	1.4581		-0.7250	-3.088	6 1.0475	-2.708	
0.7665	1.4922		-0.8106	-3.193	2 1.1703	-2.8282	
0.7968				-3.271	8 1.3014	-2.9258	
0.8271				-3.426	6 1.4365	-3.0378	
0.8577				-3.522	1.5616	5 -3.1396	

Table 3: The incremental alignment for our selected SDRB 'S with <sup>143</sup>Eu as a reference.



Rotational Frequency ħ( (MeV)



**Fig. 5.** The incremental alignments i of yrast SD bands relative to <sup>143</sup> Eu (SD1) are plotted against rotational frequency for three groups (a) N=79 isotones (b) N=80 isotones (c)N=81 isotones

#### **VI. CONCLUSION**

In this work, eight yrast SD bands observed around <sup>144</sup>Gd have been systematically studied. We used the simple three parameters rotational energy formula of Harris to suggest the unknown spins by using the experimental dynamical moments of inertia considering zero initial alignment. The inertial parameters of such formula have been adopted by using a fitting procedure. These parameters depend sensitively on how many and which members of the band are included in the fitting procedure. A good agreement between the calculated transition energies  $E_{\gamma}$ , rotational frequencies  $\hbar\omega$ , dynamic J<sup>(2)</sup> and kinematic J<sup>(1)</sup> moments of inertia and the corresponding experimental ones are obtained. Flat dynamical moments of inertia are observed for the N=80 isotones except for <sup>144</sup> Gd a band crossing at  $\hbar\omega \sim 0.45 \, MeV$  happened due to aligned of two protons in the  $i_{13/2}$  orbital, the gain in alignment is about  $6.5\hbar$ . The incremental alignment also has been calculated relative to the yrast SD band of <sup>143</sup>Eu.

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